Exam 1 Solutions

Your Name:

Instructions

Solve each of the following problems to the best of your abilities. I will give partial credit for solutions, so show all of your work.

You are allowed one 8 1/2 by 11" sheet of paper for notes as well as a calculator to aid you on the test. I can also provide you with some extra sheets of blank paper if needed.

The exam is calibrated for around 75 minutes, and you have the full class period. Once you have completed the exam, hand it to me and then you are free to leave. Good luck!

Problem 1

(32 points) Two point charges, each $Q = +100\,\mu C$, are locked in place at the points (1.0 cm, 0.0 cm) and (-1.0 cm, 0.0 cm).

- (8 points) What is the electric field at the origin?

You could use $\vec{E} = kQ/r^2\hat{r}$ to find the electric field from each point charge and then add them up as vectors at the origin. However, in this case, you have two identical point charges that are located equal distances from the origin. By symmetry, the electric fields are going to be equal and opposite. Thus, the net electric field at the origin is just $E = 0\,N/C$.

- (8 points) What is the electric potential at the origin?

The electric potential from one of the positive charges is:

$$V = \frac{kQ}{r} = \frac{(9 \times 10^9\,Nm^2/C^2)(100 \times 10^{-6}\,C)}{0.01\,m} = 9 \times 10^7\,V$$

The net electric potential is just two times that potential since potential is a scalar:

$$V_{total} = 1.8 \times 10^8\,V$$

- (8 points) If I were to place a point charge $q = -50\,\mu C$ at the origin, what is the electric force that it would feel from the two fixed point charges?

The electric force on a point charge is $\vec{F} = q\vec{E}$. Since the electric field is zero, the electric force is also zero.

- (8 points) If I were to place a point charge $q = -50\,\mu C$ at the origin, what is the electric potential energy that it would have from the two fixed point charges?

The potential energy on the negative charge at the origin is the sum of the potential energies between the negative charge and each of the positive charges:

$$U = \frac{kQq}{r} + \frac{kQq}{r} = \frac{2kQq}{r}$$

This yields:

$$U = \frac{2(9 \times 10^9\, Nm^2/C^2)(100 \times 10^{-6}\, C)(-50 \times 10^{-6}\, C)}{0.01\, m}$$

$$U = -9000\, J$$

Problem 2

(28 points) A metallic sphere with a radius of 15 cm is charged up to $+1.2\,mC$. A second, identical metallic sphere with a charge of $-0.5\,mC$ is touched to the first and then the two are separated.

- (4 points) Is this an example of charging by contact, friction, or induction?

This is charging by contact.

- (8 points) What is the final charge on each metallic sphere after they are separated?

The total initial charge was $1.2\,mC - 0.5\,mC = 0.7\,mC$. Since the spheres are identical, the charge on each will be the same after they are separated. Thus, the charge on each sphere is $0.7\,mC/2 = 0.35\,mC$.

- (8 points) What is the electric potential at the surface of each sphere after they are separated?

The potential is given by:

$$V = \frac{kQ}{r}$$

$$V = \frac{(9 \times 10^9\,Nm^2/C^2)(0.35 \times 10^{-3}\,C)}{0.15\,m}$$

$$V = 21,000,000\,V$$

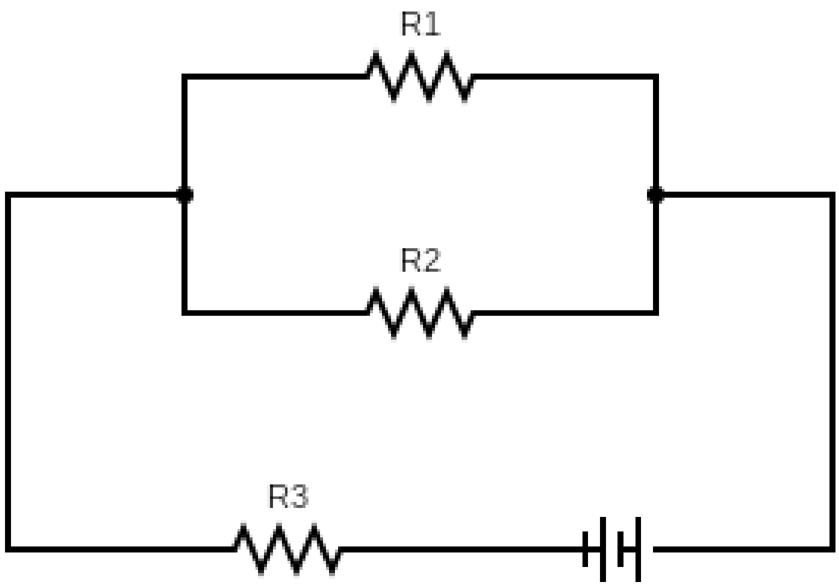
That is a lot of voltage!

- (8 points) Where does excess charge reside on a conductor?

Excess charges on a conductor reside on the surface. This is because if there were any excess charges / electric fields inside of a conductor, the charges would move due to the fields and reorient themselves to reach equilibrium. Equilibrium occurs when the net electric field inside the conductor is zero.

Problem 3

(24 points) Consider the circuit displayed below. The values of the resistors are $R_1 = 15\,\Omega$, $R_2 = 20\,\Omega$, and $R_3 = 10\,\Omega$. The battery has a voltage of 5 V.



- (8 points) What is the total resistance of the circuit?

First, we can add R_1 and R_2 in parallel. The equivalent resistance of those two resistors is:

$$\frac{1}{R_{12}} = \frac{1}{15\,\Omega} + \frac{1}{20\,\Omega}$$

$$R_{12} = \frac{60}{7}\,\Omega \approx 8.57\,\Omega$$

Then, we can add R_{12} in series with R_3:

$$R_{total} = R_{12} + R_3 = 8.57\,\Omega + 10\,\Omega = 18.57\,\Omega$$

- (8 points) What is the current flowing out of the battery?

Once we know the equivalent resistance of the circuit, we can use Ohm's law to figure out the current flowing out of the battery.

$$I = \frac{V}{R} = \frac{5\,V}{18.57\,\Omega} = 0.269\ A$$

- (8 points) If resistor R_1 were to suddenly burn out and create a break in the circuit, would you expect the current drawn from the battery to increase, decrease, or stay the same? Why?

If resistor R_1 were to suddenly burn out, the two remaining resistors - R_2 and R_3 - would be in series. The new equivalent resistance would be $R_2 + R_3 = 30\,\Omega$, which is larger than the initial resistance. Thus, the current drawn from the battery would decrease since $I = V/R$.

Problem 4

(16 points) A ceramic capacitor with a capacitance of 100 mF is charged up using a 9 V battery.

- (8 points) What is the total charge stored on the capacitor?

We can use the definition of capacitance to solve this problem:

$$Q = CV = (100\, mF)(9\; V) = 900\, mC = 0.9\, C$$

- (8 points) These capacitors are classified as ceramic because the dielectric in the capacitor is a ceramic material. What is a dielectric material and why might we use one inside of this capacitor?

A dielectric material is an electrical insulator that can be polarized by an external electric field. A dielectric inserted between the plates of a parallel plate capacitor will allow the plates to store more charge by decreasing the effective field between the plates.