Your Name:

Instructions

Solve each of the following problems to the best of your abilities. I will give partial credit for solutions, so show all of your work. I will not accept answers without any work shown.

You are allowed three 8 1/2 by 11" sheets of paper for notes as well as a calculator to aid you on the test. I can provide you with some extra sheets of blank paper if needed.

The exam is calibrated for around 90 minutes, but you have the full class period. Once you have completed the exam, hand it to me and then you are free to leave. Good luck!

Problem 1

(20 points) A series RLC AC circuit consists of a resistor (R = 10 Ω), a capacitor (C = 0.15 F), an inductor (L = 0.05 H), and a power source with an adjustable frequency (V_{max} = 12 V).

1. (5 points) What is the impedance of the circuit when it is driven at $\omega = 150\, rad/s$?

We can start by calculating the reactance of each circuit element.

$$\chi_C = \frac{1}{\omega C} = \frac{1}{(150\, rad/s)(0.15\, F)} = 0.044\, \Omega$$

$$\chi_L = \omega L = (150\, rad/s)(0.05\, H) = 7.5\, \Omega$$

$$R = 10\, \Omega$$

Then, the total impedance is given by:

$$Z = \sqrt{(\chi_L - \chi_C)^2 + R^2} = \sqrt{(7.5\, \Omega - 0.044\, \Omega)^2 + (10\, \Omega)^2} = 12.5\, \Omega$$

2. (5 points) What is the phase angle of the circuit when it is driven at $\omega = 150\, rad/s$?

We can use the reactances calculated above to calculate the phase angle:

$$tan\phi = \frac{\chi_L - \chi_C}{R} = \frac{7.5\,\Omega - 0.044\,\Omega}{10\,\Omega}$$

$$\phi = 0.64\,rad = 36.7^\circ$$

3. (5 points) What is the impedance of the circuit when it is driven at resonance?

When the circuit is in resonance, $\chi_L = \chi_C$. Thus, the impedance is just the resistance of the resistor: $Z = R = 10\,\Omega$

4. (5 points) What is the average power dissipated in the resistor when the circuit is driven at resonance?

The average power is found using the RMS current or RMS voltage:

$$P_{avg} = I_{rms}^2 R = \frac{V_{rms}^2}{Z^2} R$$

At resonance, the impedance Z is equal to the resistance R. Thus:

$$P_{avg} = \frac{V_{rms}^2}{R}$$

The RMS voltage is given by the maximum voltage divided by $\sqrt{2}$. Thus:

$$P_{avg} = \frac{V_{max}^2}{2R} = \frac{(12\,V)^2}{2(10\,\Omega)} = 7.2\,W$$

Problem 2

(10 points) An electromagnetic wave is traveling in the vacuum of space. It is red light with a wavelength of 700 nm. The magnitude of the amplitude of the electric field of the wave is 60 V/m.

1. (1 point) What is the speed of the light wave?

Light in a vacuum has a constant speed of 3×10^8 m/s.

2. (3 points) What is the wavenumber of the wave?

The wavenumber is given by $k = 2\pi/\lambda$. Thus:

$$k = \frac{2\pi}{700\,nm} = 8,975,979\,m^{-1}$$

3. (3 points) What is the magnitude of the magnetic field formed by this wave?

The magnetic field is related to the electric field by:

$$E = cB \Rightarrow B = \frac{E}{c} = \frac{60\,V/m}{3 \times 10^{8}\,m/s} = 2 \times 10^{-7}\,T$$

4. (3 points) At time t = 0 seconds, the electric field points along the +y direction and the magnetic field points along the +x direction. In which direction is the electromagnetic wave traveling?

You can use the right hand rule to solve this part - your fingers point in the direction of the electric field and then curl in the direction of the magnetic field. Your thumb then points in the direction of the velocity. The velocity of the light wave is in the $-\hat{z}$ direction.

Problem 3

(10 points) Light traveling through air is incident on a block of material at with an incident angle of $30.0°$. The angle of refraction in the material is $22.0°$. What is the index of refraction of the material? You can assume all angles are measured with respect to the normal.

This is an application of Snell's law:

$$n_1 sin\theta_1 = n_2 sin\theta_2$$

$$(1.00)sin(30.0°) = n_2 sin(22°)$$

$$0.5 = n_2(0.3746)$$

$$n_2 = 1.335$$

Problem 4

(25 points) An object with a height of 5 cm is located 12 cm in front of a diverging mirror with a focal length of magnitude |f| = 4 cm.

1. (10 points) Draw a ray diagram of the system. Be sure to include the object, the image, the mirror, the focus, and at least two rays. Try to be as accurate as possible by drawing distances to scale.

This diagram should show a diverging mirror with an object located out further than the magnitude of the focal length. We have gone through this diagram a few times (including on the mirrors and lenses homework). See page 316 of the online textbook, figure 23.32 for a ray diagram for a diverging mirror. You can also check out this link on HyperPhysics.

2. (5 points) How far is the image from the mirror?

This is an application of the thin lens equations. Remember that the focal length of a diverging mirror has a negative sign!

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{-4\ cm} = \frac{1}{12\ cm} + \frac{1}{q}$$

$$\frac{1}{-4\ cm} = \frac{1}{12\ cm} + \frac{1}{q}$$

$$-\frac{1}{3\ cm} = \frac{1}{q}$$

$$q = -3\ cm$$

3. (5 points) Is the image real or virtual?

The image formed by this diverging mirror is virtual.

4. (5 points) What is the height of the image?

The magnification of the image is:

$$M = -\frac{q}{p} = -\frac{-3\ cm}{12\ cm} = \frac{1}{4}$$

Since the height of the object is 5 cm, the height of the image is 5/4 = 1.25 cm.

Problem 5

(15 points) I set up a double slit experiment with yellow light ($f = 5.16 \times 10^{16}$ Hz) and project the interference pattern onto a screen that is located 1.25 m away from the slits. The slits have a separation of 0.10 mm.

1. (5 points) At what distance from the center of the projected pattern would I see the second maximum?

Each of the maxima of the double slit interference pattern has a location given by:

$$dsin\theta = m\lambda$$

We can use the small angle approximation in this case since the distance from the screen to the slits is about 10^4 times bigger than the distance between the slits. Thus:

$$\frac{dy}{L} = m\lambda$$

$$y = \frac{m\lambda L}{d}$$

Frequency and wavelength are related to each other through the speed of light:

$$y = \frac{mcL}{fd}$$

Plugging in the values yields:

$$y = \frac{2(3 \times 10^8\ m/s)(1.25\ m)}{(5.16 \times 10^{16}\ Hz)(0.00010\ m)} = 0.000145\ m = 0.145\ mm$$

2. (5 points) Suppose I increase the frequency of light from yellow to violet. What will happen to the pattern on the screen? Why?

If you increase the frequency of the light, the wavelength of the light will decrease. If you decrease the wavelength of the light, the distance between the maxima will decrease as well. This is because $y \propto \lambda$ in the equation from part (1).

3. (5 points) Suppose I widen the distance between the two slits. What will happen to the pattern on the screen? Why?

If you increase the distance between the two slits, the distance between the maxima will decrease. We get this from the equation used to solve part (1) - $dy/L = m\lambda$. If $m\lambda$ is held constant, then increasing d will decrease y.

Problem 6

(20 points) Answer each of the following.

1. (5 points) What are Maxwell's equations? Why are they important in the study of physics?

Maxwell's equations are a set of four equations that describe how electric fields, magnetic fields, charges, and currents relate to one another. They are important for a many reasons, but probably the most important of which is they predict the existence of electromagnetic waves.

2. (5 points) What is the electromagnetic spectrum?

The electromagnetic spectrum is the full range of frequencies of electromagnetic waves, divided up into six or seven named regions. These regions include radio waves, infrared waves, visible light, ultraviolet waves, x-rays, and gamma rays.

3. (5 points) What is total internal reflection?

Total internal reflection is a phenomenon that occurs when light in a medium with a relatively large index of refraction encounters a boundary with another medium of smaller index of refraction. If the angle of incidence is large enough, light will bounce off of the interface rather than pass through it.

4. (5 points) What is spherical aberration? Why is it important in the study of optics?

An ideal parabolic mirror would reflect every incoming, parallel light ray through one specific focus. However, the focus of a spherical mirror is slightly different depending on where the light hits the mirror. This will cause a blurring of the image. It is important in the study of optics because you need to correct for spherical aberrations in cases where you want very sharp, high quality images.