Your Name:

Instructions

Solve each of the following problems to the best of your abilities. I will give partial credit for solutions, so show all of your work. I will not accept answers without any work shown.

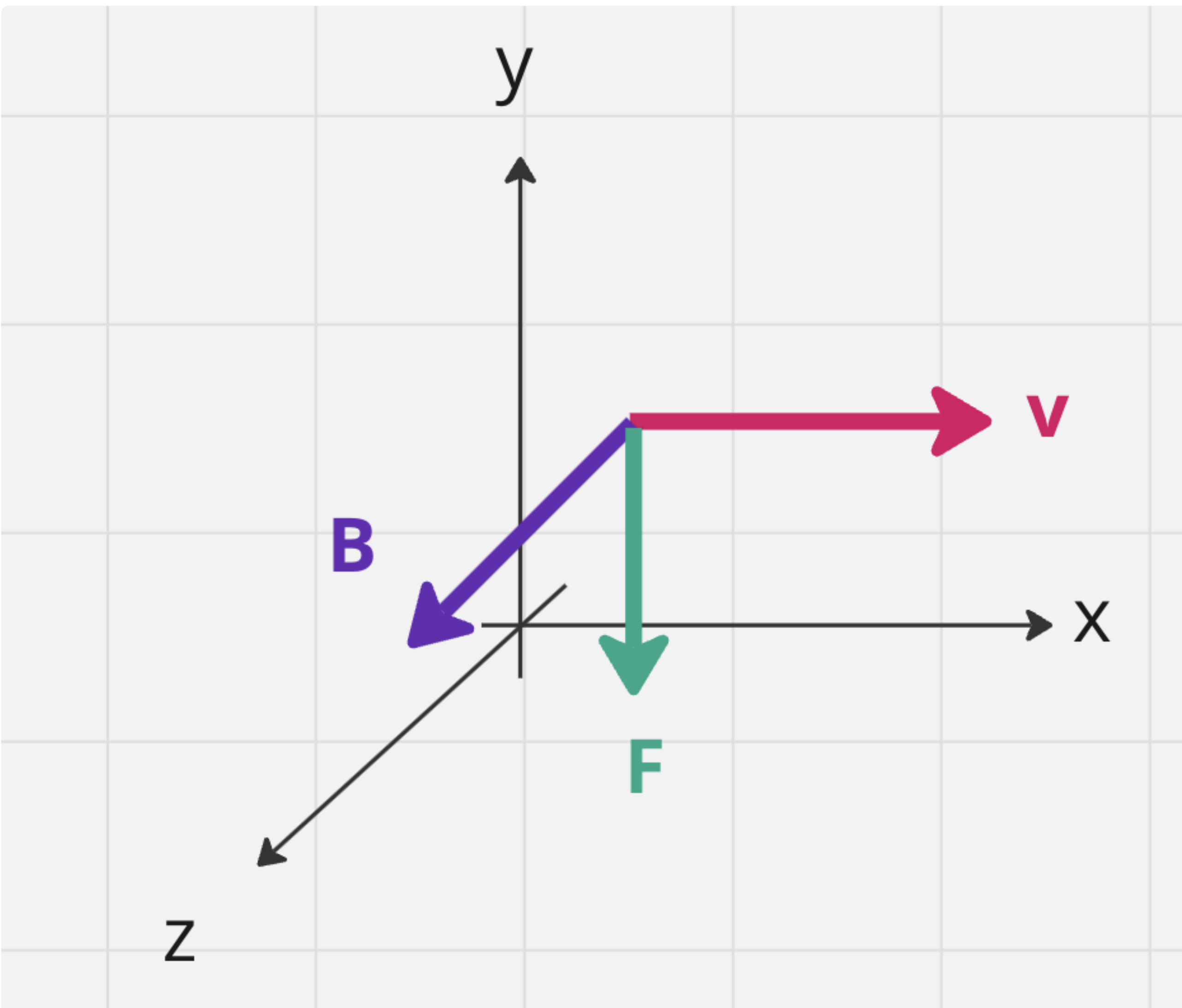
You are allowed two 8 1/2 by 11" sheets of paper for notes as well as a calculator to aid you on the test. I can provide you with some extra sheets of blank paper if needed.

The exam is calibrated for around 90 minutes, but you have the full class period. Once you have completed the exam, hand it to me and then you are free to leave. Good luck!

Problem 1

(30 points) A particle with a mass of $m = 0.22\,kg$ and a charge $q = \; +0.15\,C$ is fired into a magnetic field with a strength of $B = 0.51\,T$. The initial velocity of the particle is $\vec{v}_o = 10\,m/s$ in the $+\hat{x}$ direction while the magnetic field points in the $+\hat{z}$ direction.

1. (6 points) Draw a diagram of the system. Include and neatly label the following:
    - The directions of $\hat{x}$, $\hat{y}$, and $\hat{z}$ (remember that we use a right-handed coordinate system).
    - The particle and its initial velocity vector.
    - The magnetic field and its direction.
    - The magnetic force vector acting on the particle



2. (4 points) What is the radius of the path of the particle in the magnetic field?

Remember that the magnetic force on the particle provides the centripetal force needed to keep it in uniform circular motion. Thus, we have:

$$F_B = F_C \Rightarrow qvBsin\theta = \frac{mv^2}{r}$$

Since the magnetic field is perpendicular to the velocity vector, we know that $\theta = 90°$ and $sin\theta = 1$. Thus, we have:

$$qvB = \frac{mv^2}{r}$$
$$r = \frac{mv}{qB}$$

Plugging in values yields:

$$r = \frac{(0.22\ kg)(10\ m/s)}{(0.15\ C)(0.51\ T)} = 28.76\ m$$

3. (5 points) What is the frequency of the oscillations?

We know the radius of the oscillations is $r = mv/qB$ from the analysis above. The total circumference of the circle that the particle traverses is $C = 2\pi r = 2\pi mv/qB$. It traverses that circle with a speed v, so the time it takes to make one full revolution is $T = C/v = 2\pi m/qB$. Frequency is one over the period:

$$f = \frac{1}{T} = \frac{qB}{2\pi m} = \frac{(0.15\ C)(0.51\ T)}{2\pi(0.22\ kg)} = 0.055\ Hz$$

4. (6 points) Suppose I increase the strength of the magnetic field B, keeping everything else constant. Would the radius of the path taken by the particle increase, decrease, or stay the same? Why?

If you increase the magnetic field strength, the radius of the path would decrease since $r = mv/qB$.

5. (6 points) Suppose I invert the direction of the magnetic field B, but keep the magnitude of the field the same. Would the radius of the path taken by the particle increase, decrease, or stay the same? Why?

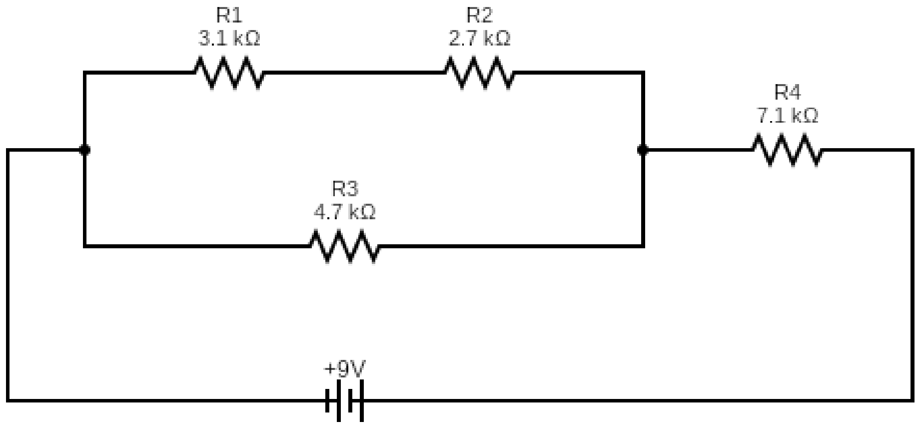
If you invert the magnetic field, the direction of the circle would change (clockwise versus counterclockwise). However, the actual radius of the circle would remain the same.

6. (3 points) Suppose I increase the initial speed of the particle keeping everything else constant. Would the frequency of the oscillations increase, decrease, or stay the same? Why?

This one is a bit unintuitive. Notice that $f = qB/2\pi m$ from the analysis above - it does not depend on the speed of the particle at all! Thus, if you increase the speed of the particle, the frequency of oscillations will remain the same.

Problem 2

(20 points) A 9 V battery is connected to a network of resistors as shown in the diagram. The resistors are $R_1 = 3.1\ k\Omega$, $R_2 = 2.7\ k\Omega$, $R_3 = 4.7\ k\Omega$, and $R_4 = 7.1\ k\Omega$.



1. (7 points) What is the equivalent resistance of the four resistors R_1, R_2, R_3, and R_4?

- We can first add R_1 and R_2 in series: $R_{12} = R_1 + R_2 = 5.8\ k\Omega$
- Next, we can add R_{12} in parallel with R_3: $R_{123} = (R_{12}^{-1} + R_3^{-1})^{-1} = 2.6\ k\Omega$
- Finally, we can add R_{123} in series with R_4: $R_{total} = R_{123} + R_4 = 9.7\ k\Omega$

2. (3 points) What is the total current drawn from the 9 V battery?

We can use Ohm's law here: $V_{total} = IR_{total}$. Thus, the current draw is $(9\ V)/(9.7\ k\Omega) = 0.9\ mA$.

3. (5 points) Would you expect the voltage drop across R_1 to be greater than, less than, or equal to the voltage drop across R_2? Why?

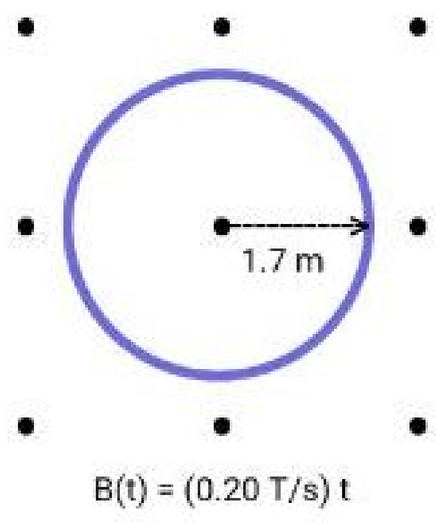
Resistors R_1 and R_2 are on the same branch, and thus, have the same current passing through them. By Ohm's law, the voltage drop across a resistor is $V = IR$. Thus, the resistor with the larger resistance (R_1) ought to have the larger voltage drop.

4. (5 points) What is conventional current and how is it different from the "actual" current in a circuit?

Conventional current is a model where positive charge carriers cause current flow. In other words, conventional current flows from the positive terminal to the negative terminal of a battery. In the actual current in a circuit, negative electrons are the charge carriers, so the flow of charges is actually in the opposite direction.

Problem 3

(30 points) A loop of wire with 21 turns and a radius of 1.7 meters is located in a magnetic field directed out of the page as shown in the diagram. The magnetic field starts at zero Teslas and increases at a rate of 0.20 Teslas per second. The wire has a total resistance of $200\ \Omega$.



1. (7 points) What is the induced EMF in the wire?

This is a classic example of Faraday's law of induction. The first step in the process is to identify whether we have a changing magnetic field, area, or angle. In this case, we have a changing magnetic field:

$$\varepsilon = -NAcos\theta\frac{\Delta B}{\Delta t}$$

Similar to how we did in lecture, I am going to solve for the magnitude of the EMF, and then use Lenz's law afterwards to solve for the direction:

$$|\varepsilon| = NAcos\theta\frac{\Delta B}{\Delta t}$$

There are 21 loops in the wire, so $N = 21$.

$$|\varepsilon| = (21)Acos\theta\frac{\Delta B}{\Delta t}$$

The area vector is parallel to the magnetic field, meaning the angle θ is zero. This means $cos\theta = 1$.

$$|\varepsilon| = (21)A\frac{\Delta B}{\Delta t}$$

The area of a circle is $A = \pi r^2$.

$$|\varepsilon| = (21)\pi r^2\frac{\Delta B}{\Delta t}$$

Plugging in values yields:

$$|\varepsilon| = (21)\pi(1.7\ m)^2(0.20\ T/s)$$
$$|\varepsilon| = 38.1\ V$$

2. (3 points) What is the magnitude of the current in the wire?

This is just an application of Ohm's law. $I = V/R = (38.1\ V)/(200\ \Omega) = 0.19\ A$.

3. (5 points) In which direction is the current flowing (clockwise or counterclockwise)?

Clockwise, due to Lenz's law.

4. (5 points) What is the power dissipated in the wire?

$$P = I^2R = (0.19\ A)^2(200\Omega) = 7.22\ W$$

5. (5 points) Suppose I started the magnetic field at five Teslas, but kept the same rate of change (0.20 Teslas per second). Would you expect the induced EMF to increase, decrease, or stay the same? Why?

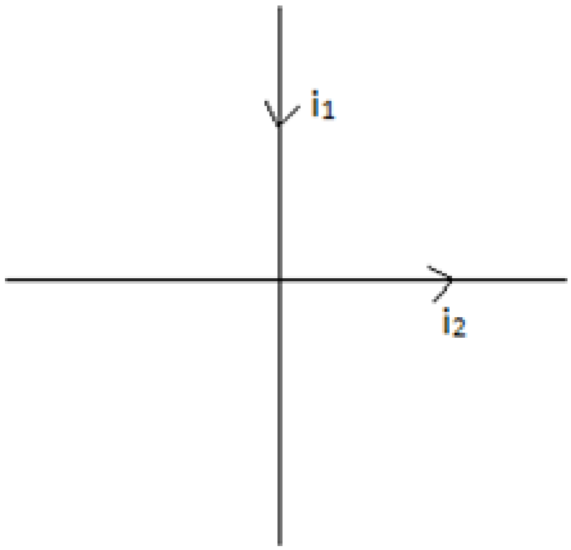
The induced EMF only depends on the rate of change of magnetic field. Thus, the starting point of the magnetic field makes no difference, and the induced EMF would stay the same.

6. (5 points) Suppose I increased the area of the loop of wire and kept everything else the same. Would you expect the induced EMF to increase, decrease, or stay the same? Why?

The induced EMF is proportional to the area of the enclosed magnetic flux. Bigger area means a bigger EMF according to Faraday's law of induction.

Problem 4

(20 points) Two crossed wires are located in the xy-plane as shown in the diagram. One carries a current of $i_1 = 1.0\ A$ in the $-\hat{y}$ direction (along the y-axis) while the other carries a current of $i_2 = 2.1\ A$ in the $+\hat{x}$ direction (along the x-axis).



1. (15 points) What is the total magnetic field from both wires at the point (2.0 m, 2.0 m)? Make sure you specify both magnitude and direction.

The magnitude of the magnetic field from a current-carrying wire is given by:

$$B = \frac{\mu_o I}{2\pi r}$$

We can find the magnetic field from each wire, use the right-hand rule to calculate its direction, and then sum them together. Recall that $\mu_o = 4\pi \times 10^{-7}$. We need to use the perpendicular distance from each wire to the point (NOT $\sqrt{2^2 + 2^2}$). For the first wire (I_1), the magnetic field is given by:

$$B = \frac{(4\pi \times 10^{-7})(1.0\ A)}{2\pi(2.0\ m)} = 1.0 \times 10^{-7}\ T$$

Its direction is out of the page (thumb points in the direction of the current, fingers curl in the direction of the magnetic field).

For the second wire (I_2), the magnetic field is given by:

$$B = \frac{(4\pi \times 10^{-7})(2.1\ A)}{2\pi(2.0\ m)} = 2.1 \times 10^{-7}\ T$$

Its direction is out of the page.

Since the directions of the magnetic fields are both out of the field, the total field is: $3.1 \times 10^{-7}\ T$ out of the page.

1. (5 points) What is Ampere's law? Why is it useful?

Ampere's law tells us that the magnetic field that circles around a current source is proportional to the strength of the current. Put another way, for any closed loop path, the sum of the magnetic field times the length elements of the loop is equal to the permeability of free space multiplied by the electric current enclosed in the loop. This allows us to calculate the magnetic field due to current carrying sources like wires.

Problem 5 - Extra Credit Challenge

(5 points) A point charge with a mass of 2.5 g and a charge of 10.5 μC has a velocity of $\vec{v} = 2.0\hat{x} + 3.0\hat{y}$ m/s. It enters a region of uniform magnetic field given by $\vec{B} = 0.5\hat{y} - 2.7\hat{z}$ T. What is the magnetic force on the particle immediately after it enters this magnetic field?

Like other examples that we have done in the past, we know that the magnetic force on the particle is given by:

$$\vec{F} = q\vec{v} \times \vec{B}$$

What makes this a challenge problem is that you have to actually do the cross product since the velocity and magnetic field vectors are not perpendicular to each other. Let's do it!

Recall the definition of the cross product that we talked about at the beginning of the semester:

$$\vec{v} \times \vec{B} = (v_y B_z - v_z B_y)\hat{x} - (v_x B_z - v_z B_x)\hat{y} + (v_x B_y - v_y B_x)\hat{z}$$

In this problem, v_z and B_x are equal to zero. Thus, we can simplify the expression above:

$$\vec{v} \times \vec{B} = (v_y B_z)\hat{x} - (v_x B_z)\hat{y} + (v_x B_y)\hat{z}$$

Let's plug in values:

$$\vec{v} \times \vec{B} = (3.0)(-2.7)\hat{x} - (2.0)(-2.7)\hat{y} + (2.0)(0.5)\hat{z} = -8.1\hat{x} + 5.4\hat{y} + 1.0\hat{z}$$

And now multiply through by the charge:

$$\vec{F} = q\vec{v} \times \vec{B} = (10.5\ \mu C)(-8.1\hat{x} + 5.4\hat{y} + 1.0\hat{z})$$

$$\vec{F} = -85.05\hat{x} + 56.7\hat{y} + 10.5\hat{z}\ \mu N$$