Your Name:

Instructions

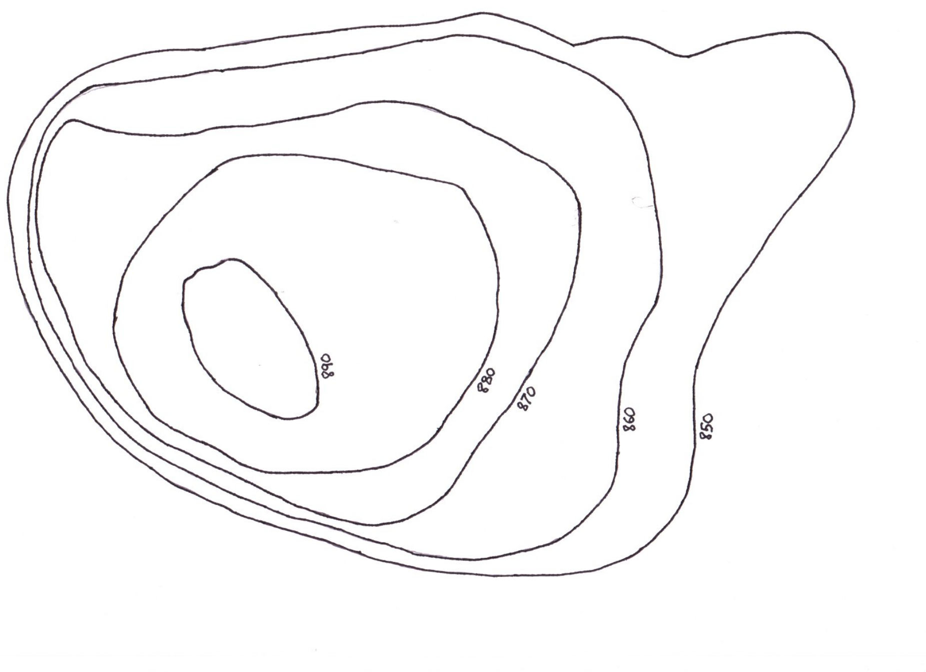
Solve each of the following problems to the best of your abilities. I will give partial credit for solutions, so show all of your work.

You are allowed one 8 1/2 by 11" sheet of paper for notes as well as a calculator to aid you on the test. I can also provide you with some extra sheets of blank paper if needed.

The exam is calibrated for around 90 minutes, but you have the full class period. Once you have completed the exam, hand it to me and then you are free to leave. Good luck!

Problem 1

(30 points) The diagram below shows the equipotential lines in some region of space. The voltage value for each equipotential line is labeled (i.e. 890 V, 880 V, 870 V, etc.).



- (10 points) Where would you expect the magnitude of the electric field to be the strongest? Make that spot directly on the diagram and explain why you chose it.

The electric field is strongest in the region where the potential lines are closest together since $E = -\Delta V/d$. If you marked any space on the left or bottom-left side of the diagram where the field lines are close together, I gave you full credit.

- (5 points) What is the direction of the electric field at the point that you chose? Mark the direction directly on the diagram.

The electric field points in the direction of decreasing electric potential. Thus, it will point outwards, away from the center of the potential lines.

- (5 points) Suppose I put a positive point charge $Q = +10\,mC$ on the 870 V equipotential line. What is its change in electric potential energy if I move it to the 850 V equipotential line?

The change in electric potential energy can be found using the change in voltage and the charge:

$$\Delta U = q\Delta V = (+10\,mC)(850\,V - 870\,V)$$
$$\Delta U = -200\,mJ$$

Note that the answer is negative since $V_{final} < V_{initial}$.

- (5 points) Suppose I put a negative point charge $Q = -10\,mC$ on the 870 V equipotential line. What is its change in electric potential energy if I move it to the 850 V equipotential line?

This is identical to what we did in the previous section, except the charge is negative. Therefore:

$$\Delta U = +200\,mJ$$

- (5 points) Suppose I put a positive point charge $Q = +10\,mC$ on the 870 V equipotential line. What is its change in electric potential energy if I move it around the 870 V equipotential line?

When you move a point charge around an equipotential line, there is no change in voltage, meaning there a change in electric potential energy. In other words, $\Delta U = 0$.

Problem 2

(35 points) I fire two helium-4 nuclei at each other in a particle collider. Each helium-4 nucleus has a mass of $6.7 \times 10^{-27}\,kg$ and a charge of $+3.2 \times 10^{-19}\,C$. They start very far apart and each have an initial speed of $17000\,m/s$.

Hint: We can use conservation of energy in this problem. Like the problems we did in class and on the exam review, you can assume that they start "infinitely" far apart.

- (5 points) What is the kinetic energy of one of the helium nuclei when they are very far apart?

We can use the equation for kinetic energy to calculate the energy of one nucleus:

$$K = \frac{1}{2}(6.7 \times 10^{-27}\,kg)(17000\,m/s)^2 = 9.6815 \times 10^{-19}\,J$$

- (5 points) What is the electric potential energy of the system when the nuclei are very far apart?

We know the $U_{electric} = kQq/r$ where r is the separation between the two point charges. If the nuclei are infinitely far apart, this means that $r \to \infty$ and $U = 0$.

- (5 points) What is the kinetic energy of one of the helium nuclei when they are at the point where they are closest together?

As the nuclei approach each other, they repel since they are both positively charged. They slow down as they approach and eventually come to a stop before reversing direction and flying away from each other. When the nuclei are closest together, their speed is briefly zero. Thus, $K = 0$.

- (10 points) Calculate the distance between the helium nuclei when they are closest together.

We have two helium-4 nuclei approaching each other, so the total initial kinetic energy is twice the kinetic energy of one nucleus and the charges are equal (call it q). Using conservation of energy, we get:

$$2K = \frac{kQq}{r} \Rightarrow r = \frac{kq^2}{2K}$$

$$r = \frac{(9 \times 10^9 \, N \cdot m^2/C^2)(3.2 \times 10^{-19} \, C)^2}{2(9.6815 \times 10^{-19} \, J)}$$

$$r = 4.759593 \times 10^{-10} \, m$$

- (5 points) Calculate the electric force between the helium nuclei when they are closest together.

The Coulomb force or electric force can be calculated using the charge given in the problem statement and the radius calculated above:

$$F = \frac{kq^2}{r^2} = \frac{(9 \times 10^9 \, N \cdot m^2/C^2)(3.2 \times 10^{-19} \, C)^2}{4.759593 \times 10^{-10} \, m}$$

$$F = 1.9363 \times 10^{-18} \, N$$

This might not seem like a lot of force, but remember, the mass of the helium-4 nucleus is $6.7 \times 10^{-27} \, kg$. Thus, the acceleration on the nucleus is around $28,900,000 \, m/s^2$!

- (5 points) Suppose I use helium-3 instead of helium-4. This means that the charge on each helium nucleus is the same, but the mass is smaller. Would you expect the distance of closest approach to be larger, smaller, or the same? Why?

Recall from above that:

$$r = \frac{kq^2}{2K} = \frac{kq^2}{mv^2}$$

I substituted in $K = \frac{1}{2}mv^2$ and simplified the expression.If the charge on each nucleus remains the same, but the mass decreases, the radius will get larger since m is in the denominator of the expression.

Problem 3

(35 points) A parallel plate capacitor is made of two plates, each with an area of $0.03\,m^2$, separated by a distance of $0.01\,m$. I connect this capacitor to a battery with a voltage of $15\,V$. The permittivity of free space is $\epsilon_o = 8.85 \times 10^{-12}\,F/m$.

- (5 points) What is the capacitance of this capacitor?

We can plug in the numbers using the formula for the capacitance of a parallel plate capacitor.

$$C = \frac{A\kappa\epsilon_o}{d}$$

$$C = \frac{(0.03\,m^2)(8.85 \times 10^{-12} F/m)}{0.01\,m}$$

$$C = 2.655 \times 10^{-11}\,F$$

- (5 points) What is the charge stored on this capacitor in the circuit?

From the definition of capacitance, we have:

$$Q = CV$$

$$Q = (2.655 \times 10^{-11}\,F)(15\,V)$$

$$Q = 3.98 \times 10^{-10}\,C$$

- (5 points) What is the energy stored on this capacitor in the circuit?

The energy stored on the capacitor is:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.655 \times 10^{-11} F)(15\,V)^2 = 2.99 \times 10^{-9}\,J$$

- (5 points) What is the magnitude of the electric field between the plates of the capacitor?

The magnitude of the electric field is given by:

$$E = \frac{\Delta V}{d} = \frac{15\ V}{0.01\ m} = 1500\ N/C$$

- (5 points) Suppose I increase the distance between the plates of the capacitor while keeping it connected to the battery. Will the voltage across the capacitor increase, decrease, or stay the same? Why?

Since the capacitor is still connected to the same battery, the voltage across it will remain constant. The physical properties of the capacitor do not affect the potential across it.

- (5 points) Suppose I increase the distance between the plates of the capacitor while keeping it connected to the battery. Will capacitance of the capacitor increase, decrease, or stay the same? Why?

Since $C = A\kappa\epsilon_o/d$, increasing the plate distance d will decrease the capacitance of the capacitor.

- (5 points) Suppose I insert a dielectric into this capacitor (you can assume $\kappa > 1$). Will the charge stored on the capacitor increase, decrease, or stay the same? Why?

Since $C = A\kappa\epsilon_o/d$, adding a dielectric will make the capacitance increase since κ will now be a number greater than one.